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All Eulerian method of computing elastic response of explosively pressurised metal tube

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We present an all Eulerian approach to simulate the elastic response of a metal tube loaded explosively by a gaseous detonation. The high strain rate deformation of the metal tube subjected to high explosive detonation is mathematically described by hyperbolic processes where the characteristics of existing wave motions were correlated with the local particle velocities through the speed of sound in the metal. This is a favourable case for the hydrocode which is based on a compressible gas dynamics solver and for simulating a high strain rate and dominantly plastic response of a material subject to an explosive loading. The hydrocodes fall substantially short of predicting elastic motion without the plastic flow of the confining material, for relatively minor pressure loadings due to a gaseous explosion as opposed to a high explosive detonation of a charged tube. The corresponding loading pressure due to gaseous explosion is a few orders of magnitude lower than those resulting from high explosive loadings. Utilising a hydrocode designed to handle the reactive process leading to a plastic flow of the confining materials is of great interest and a significant challenge. The new technique, based on the Eulerian framework, preserves the feature of a Lagrangian code while utilising all the benefits of an Eulerian solver that uses fixed grids with the level-sets for defining the multi-material interfaces. The hybrid particle level-set algorithm is combined with a hydrodynamic solver that adds an elasticity correction when handling the structural response while the overall scheme remained hyperbolic during the entire reactive flow. Several unseen dynamics of detonation flow associated with the elastically loaded tube of finite thickness are reported by using the present method for analysing the highly pressurised vessel.

Keywords: Eulerian; detonation; elastic; elasto-plastic; multi-material

1. Introduction

Detonation, commonly known as abnormal combustion, has both positive and negative aspects of a rapid chemical reaction induced by the potentially high thermodynamic attribute. When properly utilised, gaseous detonation of a hydrocarbon mixture generates thrust in a pulse detonation engine (PDE), which is a potentially highly efficient engine for aircraft propulsion [1]. However, unintended detonation, accompanied by a structural deformation or failure, can cause serious accidents such as a pipe rupture leading to a catastrophic disaster. The structural response to an internal gaseous detonation is unknown as the thermal flow will be affected by the elastic pulsation of a tube of finite thickness.

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The unstable deflagration may be accelerated to a detonation or may decelerate to a weak deflagration. An exact theory of such complex elastically vibrating wall effects is unknown and at best can be understood from direct numerical simulation. In order to accurately predict and reproduce this phenomenon, a sound numerical technique must be selected for handling the high strain dynamics of a detonation flow as well as the elastic behaviour of weakly vibrating, and strongly rupturing confinement tubes. An elegantly uncomplicated technique for the simulation of structural response, and flow detonation follows a unified approach, formulated in an all Eulerian framework including the elastic recovery function.

For analysis of multi-material dynamics, various numerical approaches including Lagrangian, Eulerian, and arbitrary Lagrangian–Eulerian (ALE) methods have been developed. A fully Lagrangian method has advantages in the simulations of structural deformations that include elastic motion, crack, and damage of structure [2,3]. When the deformation is relatively large, however, the computational grids become significantly distorted and may even disappear completely, leading to a drop in the intended numerical accuracy. Furthermore, the method falls short in capturing the shock waves present in the general multi-material problem. A family of Eulerian schemes [4–6] is currently evolving into a maturing state where the flow quantity is considered at a fixed point. The interface tracking is handled by the hybrid particle level-set (HPLS) method while the ghost fluid method (GFM) is used to track the location of the contact boundary and the free surface, where there exists the need for replacing real nodes with ghost nodes for prescribing the accurate isentropic condition across the interface. Also, large deformation of solids is easily handled. However, any weak perturbations and minor deformation of the elastically deforming material interface are not captured via the standard Eulerian method.

The ALE method combines best of both worlds by allowing Lagrangian grids to behave as Eulerian grids when large deformation is anticipated in the flow [7–9], but the method is preferred only if all the grids are either fully Lagrangian or fully Eulerian. An ideal ALE is still under development to meet the needs of the high pressure multi-material dynamics. Nonetheless, the weakness of Lagrangian method when calculating shock wave physics [10] and those of the Eulerian method for complimenting correct modification of multi-material tracking [11–14] have been addressed.

In the authors’ previous works [15,16], we considered the plastic flow of a tube associated with high pressure loading by making full use of the Eulerian method. Here, we noted that any elastic response of the tube could not be precisely captured by the proposed method as there was no elastic recovery feature implemented into the formulation of the Eulerian scheme.

The currently available literature on the problem of a detonation-loaded tube structure treats the structure as rigid [17,18] or plastically deforming. The case when detonation gives rise to a minute oscillation of the walls which generates perturbation of weak deflagration has not been addressed properly.

In this paper, based on a fully Eulerian formulation, the numerical simulation of gaseous mixture detonation within a steel tube is conducted by utilising (i) the detonation reaction models of ethylene–oxygen and kerosene–air mixtures; (ii) the elasto-plastic problem validated via the Taylor impact; and (iii) purely elastic vibration response of metal [19–21]. Immediate use of the proposed method may be in furthering understanding PDE operation with the operating frequency of approximately 100 Hz [22] as it could further vibrate at its natural frequencies.
2. Numerical model

2.1 Governing equations

Our study presents the Eulerian hydrodynamics approach formulated in a rectangular coordinate system to simulate the dynamic interactions between the detonation wave and the metal undergoing elasto-plastic response. In order to simulate the response of a metal, one considers the elastic deformation whose work must not influence the entropy evolution. Thus the specific total energy, $e$, can be modified to include the elastic contribution, such that:

$$e = e_i + e_k + e_E$$  \hspace{1cm} (1)

$$\frac{de_E}{dt} = \frac{1}{\rho} \left( S_{ij} : D_{ij}^E \right)$$  \hspace{1cm} (2)

where $e_i$, $e_k$, $e_E$, $S_{ij}$, and $D_{ij}^E$ are the specific internal energy, specific kinetic energy, specific elastic energy, deviatoric stress tensor, and elastic strain rate tensor, respectively. The specific elastic energy, $e_E$ is associated with the elastic work, and its rate of change is expressed as Equation (2). In general shock physics analysis, the elastic energy is ignored in calculating the total energy. This is because the elastic energy, $e_E = S_{ij} : S_{ij} / 4 \rho G$ is relatively smaller than the rest of the components [20,23]. However, a realistic simulation should include $S_{ij} : S_{ij} / 4 \rho G$ and such $e_E$ is considered in the present energy conservation.

The following conservative laws of mass, momentum, and energy in an axisymmetric cylindrical ($\alpha = 1$, r- and z-axis) and rectangular ($\alpha = 0$, x- and y-axis) coordinates are used for metal. Also the deviatoric stress, $S_{ij}$ is calculated together with the evolution equations based on a Hooke’s law and the plasticity flow theory for the high strain rate deformation:

$$\frac{\partial U}{\partial t} + \frac{\partial G}{\partial r} + \frac{\partial F}{\partial z} + H = 0$$  \hspace{1cm} (3)

$$U = (\rho, \rho u_r, \rho u_z, \rho (\rho e + P), S_{rr}, S_{zz}, S_{rz})^T$$

$$G = (\rho u_r, \rho u_r^2 + P, \rho u_r u_z, \rho (\rho e + P), S_{rr}u_r, S_{zz}u_r, S_{rz}u_r)^T$$

$$F = (\rho u_z, \rho u_r u_z, \rho u_z^2 + P, u_z (\rho e + P), S_{rr}u_z, S_{zz}u_z, S_{rz}u_z)^T$$

$$H = \begin{pmatrix}
\alpha \frac{\rho u_r}{r} - \frac{\partial S_{rr}}{\partial r} - \frac{\partial S_{rz}}{\partial z} + \frac{\partial}{\partial t} (\rho u_{E,r})

\alpha \left( \frac{\rho u_r}{r} - \frac{\partial S_{rr}}{\partial r} - \frac{\partial S_{rz}}{\partial z} + \frac{\partial}{\partial t} (\rho u_{E,r}) \right) - \frac{\partial (u_r S_{rr} + u_z S_{rz})}{\partial r} - \frac{\partial (u_r S_{zz} + u_z S_{rz})}{\partial z} - 2G \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} - \eta D_{rr}^P \right)

-2S_{rz} \Omega_{rz} - S_{rr} \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} \right) - 2G \left( \frac{\partial u_z}{\partial z} - \eta D_{zz}^P \right)

2S_{rz} \Omega_{rz} - S_{zz} \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} \right) - 2G \left( \frac{1}{2} \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} \right) - \eta D_{rr}^P \right)

-\Omega_{rz} (S_{zz} - S_{rr}) - S_{rz} \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} \right) - 2G \left( \frac{1}{2} \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} \right) - \eta D_{rr}^P \right)
\end{pmatrix}$$

where $\rho$, $u_r$, $u_z$, and $P$ are density, r-axis velocity, z-axis velocity, and pressure, respectively. The parameter, $\eta$ equals to 0 (or 1) in the elastic (or plastic) state. In these equations, $\Omega_{rz} = (\partial u_r / \partial z - \partial u_z / \partial r) / 2$, $G$, $\Sigma = (\partial u_r / \partial r + u_r / r + \partial u_z / \partial z) / 3$, and $D_{ij}^P$, are spin tensor, shear modulus, volume strain rate, and plastic strain rate tensor, respectively [4]. The
plastic strain rate tensor is derived by Equation (5) which satisfies the physical constraint using the radial return algorithm [24].

\[
D_{ij}^p = \frac{d}{dt} \left( \frac{\sigma - \sigma_y}{\sqrt{6G (1 + \frac{h}{3\sigma})}} \right) \frac{S_{ij}}{\sqrt{S_{kl}S_{kl}}} 
\]

Also, to introduce \( e_E \), in these equations, one redefines kinetic energy to include the portion of elastic velocity (\( u_{E,i} \)). In other words, the kinetic energy has the same amount of elastic energy as shown in Equation (6).

\[
\frac{\rho \ddot{u}_E \cdot \ddot{u}_E}{2} = \rho e_E = \int (S_{ij} : D_{ij}^F) \, dt 
\]

Here, the magnitude of elastic velocity vector can be derived. The unit vector of the elastic velocity, \( \frac{\dddot{a} - \dddot{b}}{|\dddot{a} - \dddot{b}|} \), is calculated from \( \dddot{a} \) and \( \dddot{b} \) which are initial and shifted normal vectors from each node to interface, respectively (see Figure 1). Therefore, at an arbitrary node in solid, the elastic velocity vector is defined and considered through momentum equations.

\[
\dddot{u}_E = \frac{\dddot{a} - \dddot{b}}{|\dddot{a} - \dddot{b}|} \cdot \dddot{u}_E = \frac{\dddot{a} - \dddot{b}}{|\dddot{a} - \dddot{b}|} \cdot \left[ \frac{2}{\rho} \int (S_{ij} : D_{ij}^F) \, dt \right]^{\frac{1}{2}} 
\]

The governing equations are solved by a third-order Runge–Kutta (RK), and the convex essentially non-oscillatory (CENO) method in the temporal and spatial discretisations, respectively.

### 2.2 Constitutive relations

To describe the elasto-plastically deforming solid, we use the Mie-Gruneisen EOS which is related to specific volume and internal energy. The current yield strength, \( \sigma_Y \), is determined by the rate-dependent Johnson–Cook strength model in which the yield stress depends on
strain rate and temperature as shown in Equations (8) and (9):

\[
P = \begin{cases} 
\rho_0 C_0^2 \mu \left[ 1 + \left( \frac{1}{\Gamma_0} - \frac{a_0 \mu}{\Gamma_0} \right) \frac{a_0 \mu^2 \mu - a_0 \mu}{\mu - a_0 \mu} \right] + (\Gamma_0 + a_0 \mu) E & \text{for } \mu \geq 0, \mu = \frac{\rho}{\rho_0} - 1 \\
\rho_0 C_0^2 \mu + (\Gamma_0 + a_0 \mu) E & \text{for } \mu < 0
\end{cases}
\]

Equation (9)

\[
\sigma_Y = \left( \sigma_{Y,0} + A \left( \frac{\dot{\varepsilon}^P}{\dot{\varepsilon}_0} \right)^n \right) \left( 1 + B \ln \left( \frac{\dot{\varepsilon}^P}{\dot{\varepsilon}_0} \right) \right) \left( 1 - \left( \frac{T - T_0}{T_m - T_0} \right)^m \right)
\]

where \( \Gamma_0, S_0, C_0, A, B, \) and \( n \) are material constants. \( \rho_0, E, T_m, T_0, \) and \( \dot{\varepsilon}_0 \) are initial density, specific internal energy, melting temperature, ambient temperature, and effective plastic strain rate respectively. The value of \( \dot{\varepsilon}_0 \) is set to unity.

2.3 Interface treatment and boundary conditions

In order to track the interface between the gaseous mixture and the metal in a fixed rectangular coordinate domain, a modified hybrid particle level-set (HPLS) method is used instead of a simple level set method as given by Equation (10) [25]:

\[
\frac{\partial \phi}{\partial t} + u_i \cdot \nabla \phi = 0
\]

Equation (10)

The interface is marked as the points of zero level set (\( \phi = 0 \)) as the region of \( \phi < 0 \) indicates the inner side of a material. Fifth order weighted ENO and third order RK method are used in the spatial and temporal derivatives near the interface (\( \phi = 0 \)) based on the particle velocities, \( u_i \), from the momentum balance. When a rapid change in the material properties distorts the interface, a periodic re-initialisation is used to recover the correct solution. The resulting re-initialisation function is defined as follows:

\[
\phi_t + S(\phi) (|\nabla \phi| - 1) = 0, \quad S = \frac{\phi}{\sqrt{\phi^2 + (1 - |\nabla \phi|)^2} \Delta x^2}
\]

Equation (11)

Nevertheless, the re-initialisation can lead to an unintentional event such as interface rounding or mass losses. In order to avoid these known problems and to obtain a precise interface solution, a modified HPLS method including particle reseeding is adapted [23]. In this method, two types of massless particles including positive and negative particles are placed in the regions \( \phi > 0 \) and \( \phi < 0 \), respectively. These particles are allowed to flow in a Lagrangian sense such that:

\[
\frac{d\vec{x}_p}{dt} = \vec{u}(\vec{x}_p)
\]

Equation (12)

where \( \vec{x}_p \) is the position of the particles and \( \vec{u}(\vec{x}_p) \) is its velocity. Although each particle has no mass, they have volume. In two dimensions, the radii of those particles are determined based on the size of the grid, and the minimum and maximum values are \( r_{\min} = 0.1 \min(\Delta r, \Delta z) \) and \( r_{\max} = 0.5 \min(\Delta r, \Delta z) \), respectively. The third order RK method is used to solve the temporal derivative. If the escaped particles with respect to the interface are detected, the error correction through a local level set reconstruction is performed.
Table 1. Material properties of copper, beryllium, and steel (304 stainless steel).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial density, ρ₀ (kg m⁻³)</td>
<td>8930</td>
<td>1845</td>
<td>7900</td>
</tr>
<tr>
<td>Initial temperature, T₀ (K)</td>
<td>293</td>
<td>293</td>
<td>293</td>
</tr>
<tr>
<td>Specific heat, cᵥ (J kg⁻¹ K⁻¹)</td>
<td>368</td>
<td>1825</td>
<td>500</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.35</td>
<td>0.053896</td>
<td>0.35</td>
</tr>
<tr>
<td>Shear modulus, G (GPa)</td>
<td>45</td>
<td>153</td>
<td>77.5</td>
</tr>
<tr>
<td>Gruneisen coefficient, Γ₁₀</td>
<td>2.0</td>
<td>2.0</td>
<td>1.93</td>
</tr>
<tr>
<td>Normal sound speed, c₀ (m s⁻¹)</td>
<td>3940</td>
<td>12870</td>
<td>4570</td>
</tr>
<tr>
<td>S₀</td>
<td>1.49</td>
<td>1.124</td>
<td>1.49</td>
</tr>
<tr>
<td>a₀</td>
<td>0.47</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Initial yield strength, σᵧ₀</td>
<td>90 MPa</td>
<td>10 GPa</td>
<td>110 MPa</td>
</tr>
<tr>
<td>A</td>
<td>292 MPa</td>
<td>–</td>
<td>1.5 GPa</td>
</tr>
<tr>
<td>B</td>
<td>2.5 × 10⁻²</td>
<td>–</td>
<td>1.4 × 10⁻²</td>
</tr>
<tr>
<td>n</td>
<td>0.31</td>
<td>–</td>
<td>0.36</td>
</tr>
<tr>
<td>m</td>
<td>1.09</td>
<td>–</td>
<td>1.0</td>
</tr>
<tr>
<td>Melting temperature, Tₘ (K)</td>
<td>1351</td>
<td>1560</td>
<td>1689</td>
</tr>
</tbody>
</table>

At the interface between materials, the boundary conditions need to be determined because of the discontinuity in entropy distribution. A GFM is used to address the multi-material problem. In this method, ghost nodes are distributed on the outside of a target material of interest using an extrapolation on the basis of the continuous entropy assumption. Here, the real discontinuity in the entropy merged with the ghost nodes generating the proper boundary conditions, and the same pressure and velocity conditions are imposed in the ghost nodes. The entropies in the ghost nodes are obtained from the real material while the remaining variables are determined from the entropy relation and the equation of state.

3. Results and discussion

A series of validation calculations for two independent detonation models of ethylene–oxygen and kerosene–air mixtures have been covered previously [15,16]. In conjunction with the detonation models, we describe the elastic and elasto-plastic responses of metal confinements, subjected to detonation loading. The initial conditions and material properties are shown in Table 1 for copper, beryllium, and 304 stainless steel.

3.1 Plasticity – Taylor test of a copper rod

In order to confirm the plastic behaviour of metal (copper) listed in Table 1, Taylor impact problem is considered as a variant of the example considered in [19] based on a two-dimensional cylindrical coordinate. The schematic of the problem is shown in Figure 2. The top, right, left, and bottom boundary conditions are zero gradient, wall, zero gradient, and axisymmetric conditions respectively.

An impact of a rod is described by prescribing an initial particle velocity which is 189 m s⁻¹. Figure 3 shows the histories of kinetic, internal, and total energy densities, and indicates that the kinetic energy gets fully converted to internal energy under a total energy conservation condition. The deformation stops at approximately 80 µs after the impact on a rigid wall.
Figure 2. 2D cylindrical rod set-up for plastic test.

Figure 3. Time history of total, kinetic, and internal energy densities under Taylor impact.

Figure 4 and Table 2 show the comparison between the experimental data and the numerical results. The initial calculation error was 6–10% (33 µs), and with time the error declined to 0.4–4%.

3.2 Elasticity – vibrating beryllium plate tests
The elasticity test consists of a vibrating two-dimensional rectangular plate of beryllium with no support at either ends. The plate is placed in two distinct surrounding mediums: (i) in void having zero constraint; and (ii) in heavy gas medium. As shown in Figure 5, the plate dimension is 60 mm × 10 mm, embedded in the calculation domain of 70 mm × 30 mm, identical to those considered in [20]. In order to enforce the elastic oscillations of the plate, a high yield strength is used. The plate is prescribed with an initial y-axis velocity
distribution as:

\[ u_y(x, y) = A \sin \left( \frac{x - 0.025}{0.03} \pi \right) \text{ m/s} \] (13)

Among the two types of surroundings considered, the free void is used to verify the elastic oscillation of the plate with an amplitude of \( A = 100 \text{ m s}^{-1} \). As shown in Figure 6, the frequency of y-axis velocity at the centre point is approximately 33 kHz, which is consistent with the FEM result using NASTRAN, 33198 Hz [26] at first bending mode.

Table 2. Comparison between experimental data and simulation results.

<table>
<thead>
<tr>
<th>Time ((\mu s))</th>
<th>Experiment [19]</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length (mm)</td>
<td>Bottom radius (mm)</td>
</tr>
<tr>
<td>33</td>
<td>51.34</td>
<td>7.02</td>
</tr>
<tr>
<td>63</td>
<td>46.87</td>
<td>7.68</td>
</tr>
<tr>
<td>80</td>
<td>44.89</td>
<td>7.68</td>
</tr>
</tbody>
</table>
Figure 5. 2D elastic test set-up with four pressure gauges.

Figure 6. Vibrating plate in void: comparison between NASTRAN and calculated y-axis velocity at centre point.

Figure 7 shows the dynamic plate response using velocity magnitude, $V = u_x^2 + u_y^2$ and deviatoric stress, $S_{rr}$. With time, the fields of deviatoric stress show the repeated compression and expansion in the top and bottom of the plate (see Figure 7(b)).

Next, we consider the elastic motion of a plate surrounded by a heavy gas, which acts as a motion damper. Figure 8 shows the central y-axis velocity shown with the void case. Driven at the same frequency as the void surrounding, the vibration amplitude is damped by the heavy surrounding gas. After approximately 0.25 ms, the velocity amplitude approaches zero as shown in Figure 9. Additional amplitudes, namely $A = 10, 50, 100$ are considered in Figure 9, where it shows the vibration frequencies for each case along with the velocity dissipation.
In order to investigate the dynamic interaction of plate and gas mixture, the velocity magnitude and stress field shown in Figure 7 are drawn in Figure 10. The velocity magnitude of the plate in heavy gas is nearly halved following its first bending motion at 23 µs. The sign of deviatoric stress in the lower part of plate is minus due to compression, whereas the upper part is a plus due to expansion. At the same time, in the surrounding gas below...
Figure 9. Histories of y-axis velocity at centre point in various initial velocity magnitude.

Figure 10. Snapshots of (a) velocity magnitude (unit: m s\(^{-1}\)) and (b) deviatoric stress, \(S_{\text{rr}}\) field inside of plate and pressure field in gas mixture.
the plate, the pressure decreases about 20% from the initial pressure 0.1013 MPa because of the expansion of the gas induced by the rapid plate compression. This drop in pressure leads the expansion wave into the opposite direction of the plate. On the other hand, in the surrounding gas above the plate, the pressure increases about 20% owing to a compression of the gas induced by the plate expansion. This pressure jump leads the compression wave into the other direction. In time, the stress distribution in the plate is reversed by the elastic motion of plate (at 23 $\mu$s). Also it is observed that expansion (compression) reverses back to compression (expansion) in the surrounding heavy gas. This transition can be confirmed by time histories of pressure taken at four locations in Figure 5. P1, P2, P3, and P4 are located in 3.3, 6.7, 23.3, and 26.6 mm from the bottom of the calculation domain, respectively. In Figure 11, pressure fluctuations waned with time and pressure recovered the stable ambient pressure of 0.1013 MPa at 0.46 ms.

3.3 Detonation of ethylene–oxygen mixture in an elasto-plastic tube

After validating both purely plastic and elastic responses of metal subject to external stimuli, a combined dynamic response of metal subject to a detonation loading is studied. First, we take a closer look at the elasto-plastic deformation of a tube which is loaded by an internal gaseous detonation. The detailed loading condition of the ethylene–oxygen gas mixture experiment is adapted from [15]. Figure 12 compares experiment and calculations of the residual strains for an initial condition (2 bar).

The purely plastic result seems broader than both experimental and elasto-plastic results. This is because the purely plastic model takes into account the liquefied solid phase following a plastic deformation. The elasto-plastic result seems to reproduce the measurement because the elastic energy and velocity give rise to an elastic recovery after the unloading of detonation.

3.4 Detonation of kerosene–air in a purely elastic tube

Figure 13 depicts a two-dimensional section of the tube (4 mm × 30 mm) shown with a thickness, $t = 0.2$ mm. The boundary conditions on the bottom, top, right, and left are
symmetric, zero gradient, zero gradient, and extrapolated $Y_{boundary} = 0.95Y_1 + 0.05Y_0$, respectively. For initiating a detonation, the C-J values are prescribed at the inlet as a plane shock wave.

The propagation of kerosene–air mixture detonation [16] under vibrating tube with natural frequency is considered. The natural frequency of the tube shown in Figure 13 is approximately 31 kHz (the first longitudinal mode and the second radial wave mode), as previously obtained from the Rayleigh method [27]. Thus, we consider a vibrating steel tube at frequency of 31 kHz with different maximum strains of 0.000125 and 0.00025 which fall within the elastic strain range.

Figure 14 shows the pressure histories of detonation for rigid tube and strong/weak vibrating tubes along the centre line. Although the detonation velocity for these tubes are measured to be the same, the pressure for vibrating tube fluctuated more. Furthermore, the perturbation of detonation is increased in accordance with increase of maximum strain. In other words, the elastic vibration of the tube disturbs the detonation front as the development
of small radial velocities and the formation of pressure gradients along the wall give rise to strong enhancement of the acoustic wave interaction.

These are consistent with the findings represented in Figure 15 which show the density contours in the rigid and strongly vibrating tubes at 8 and 14 µs. In the elastic tube, the flow field is disturbed as the winding detonation front and acoustic waves clearly appear in the burned regions, all of which are representative of real pipe flow at high pressure.
4. Conclusion

In order to incorporate the elastic response into the numerical simulation of a gaseous detonation tube, the elastic recovery is introduced into the Eulerian based hydrocode. The new method is suitable for understanding the dynamic state of pulse detonation mode associated with the small-sized tubes. Both safety and operation efficiency of a PDE can be analysed through the application of the present method.

Disclosure statement

No potential conflict of interest was reported by the authors.

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